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M.Sc. IVth SEMESTER EXAMINATION, 2019

MATHEMATICS

Paper – III

DSE – 3 {Discrete Mathematics}

Time: Three Hours

Maximum Marks: 80

PART – A (खण्ड – अ) *[Marks: 20]*

*Answer all questions (**50** words each).*

All questions carry equal marks.

*सभी प्रश्न अनिवार्य हैं। प्रत्येक प्रश्न का उत्तर **50** शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – B (खण्ड – ब) *[Marks: 40]*

*Answer **five** questions (**250** words each),*

selecting one from each unit. All questions carry equal marks.

*प्रत्येक इकाई से **एक–एक** प्रश्न चुनते हुए, कुल **पाँच** प्रश्न कीजिए।*

*प्रत्येक प्रश्न का उत्तर **250** शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – C (खण्ड – स) *[Marks: 20]*

*Answer any **two** questions (**300** words each).*

All questions carry equal marks.

*कोई **दो प्रश्न** कीजिए। प्रत्येक प्रश्न का उत्तर **300** शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – A

Q.1 Answer all questions -

(i) Statement of Handshaking lemma.

(ii) Define path, circuit with example.

(iii) A plane simple graph has 30 vertices, each of degree 3. In how many regions can this graph be partitioned?

(iv) Define cut-sets.

(v) What is post order tree traversal?

(vi) Define strong connectivity of a graph.

(vii) Define FSM.

(viii) Define Acceptor.

(ix) Define context sensitive grammar.

(x) Write statement of Pumping Lemma.

PART – B

UNIT – I

Q.2 (a) Draw the following graph –

$V = \{v_1, v_2, v_3, v_4\}$

$E = \{(v_1, v_2), (v_1, v_4), (v_2, v_3), (v_2, v_4), (v_3, v_4)\}$

then verify first theorem of graph theory.

(b) Define Induced sub-graph, Pseudo graph, Edge connectivity of a graph.

Q.3 (a) Define Planar graph. Show that complete graph K_5 is not a planar graph.

(b) Prove – The maximum number of edges in a simple graph with n vertices is $\frac{1}{2}n(n-1)$.

UNIT – II

Q.4 Define Complete Bipartite graph with suitable example. Show that if $G = (V, E)$ is a Bipartite graph with n – vertices, then the total number of edges in G can't exceed $\frac{1}{4}n^2$.

Q.5 (a) Prove – Let G be an acyclic graph with n – vertices and k connected components i.e. $\omega(G) = k$, then G has (n-k) edges.

(b) Write a note on Kruskal's Algorithm.

UNIT – III

Q.6 Prove – For any connected planar graph –

$v - e + r = 2$, where v, e and r are the number of vertices, edges & regions of the graph respectively.

Q.7 Define Directed trees. Explain search trees by using your own example.

UNIT – IV

Q.8 What are equivalent machines? Minimize the FSM M given by the following state table –

State	Input		Output
	i_1	i_2	
s_0	s_1	s_2	0
s_1	s_5	s_3	0
s_2	s_6	s_4	0
s_3	s_7	s_2	0
s_4	s_1	s_5	1
s_5	s_3	s_7	0
s_6	s_4	s_1	0
s_7	s_1	s_2	1

Q.9 Draw the transition diagram of FSM which is represented by following state table. Also find the output words corresponding to input words –

(i) w = 10111

(ii) v = 10011011

State	Transition function (f)		Output function (g)	
	0	1	0	1
s_0	s_3	s_1	0	1
s_1	s_0	s_1	0	1
s_2	s_3	s_1	0	1
s_3	s_1	s_3	0	0

UNIT –V

Q.10 Construct a grammar for the following language –

$L = \{a^i b^i \mid i, j \geq 1, i \neq j\}$

Q.11 Define Regular Expressions –

Find the language generated by the grammar

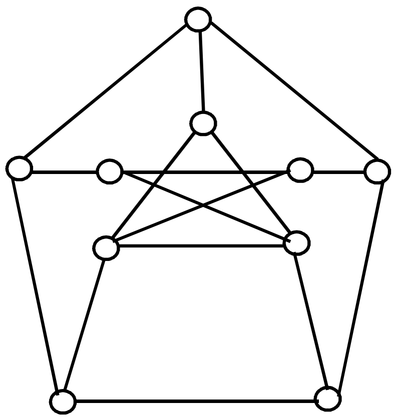
G = (V, T, S, P) where V = {S, A, B} and T = {a, b, c} and production set $P = \{S \rightarrow aSA, S \rightarrow aB, A \rightarrow b, B \rightarrow c\}$.

PART – C

Q.12 (a) Prove – If a graph (connected or disconnected) has exactly two vertices of odd degree, there must be a path joining these two vertices.

(b) Prove – A simple graph (i.e. a graph without parallel edges or self-loops) with n vertices and k components can have at most $\frac{1}{2}(n - k)(n - k + 1)$ edges.

Q.13 Is the given Petersen's graph planar? Explain your answer.



Q.14 Define Eulerian path, Eulerian circuit.

Prove – A given connected graph G is an Euler graph iff all vertices of G are of even degrees.

Q.15 Construct a Finite Automaton M with $I = \{i_1, i_2\}$ which will only accepts those strings in i_1 and i_2. Such that the number of i_1's is even and the number of i_2's is divisible by 3.

Q.16 State and prove Pumping Lemma. Show that the following language is not regular

$L = \{a^P \mid P \text{ is prime}\}$
