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M.Sc. IInd SEMESTER EXAMINATION, 2019

MATHEMATICS

Paper – IVth

Mechanics - II

Time: Three Hours

Maximum Marks: 80

PART – A (खण्ड – अ) *[Marks: 20]*

*Answer all questions (**50** words each).*

All questions carry equal marks.

*सभी प्रश्न अनिवार्य हैं। प्रत्येक प्रश्न का उत्तर **50** शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – B (खण्ड – ब) *[Marks: 40]*

*Answer **five** questions (**250** words each),*

selecting one from each unit. All questions carry equal marks.

*प्रत्येक इकाई से **एक–एक** प्रश्न चुनते हुए, कुल **पाँच** प्रश्न कीजिए।*

*प्रत्येक प्रश्न का उत्तर **250** शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – C (खण्ड – स) *[Marks: 20]*

*Answer any **two** questions (**300** words each).*

All questions carry equal marks.

*कोई **दो प्रश्न** कीजिए। प्रत्येक प्रश्न का उत्तर **300** शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – A

Q.1 Answer all questions -

(i) Write product of inertia of an elliptic quadrantal disc with respect to its axes.

(ii) Define 'Radius of gyration'.

(iii) What is principle of Angular Momentum?

(iv) State D'Alembert's principle.

(v) What is rolling and sliding friction?

(vi) Write Kinetic Energy of a rigid body in a two-dimensional motion.

(vii) Define conservative forces.

(viii) What is principle of conservation of linear momentum?

(ix) What is Lagrangian Function?

(x) What do you mean by conservative or non-conservative dynamical systems?

PART – B

UNIT – I

Q.2 Find the product of inertia of a semi-circular wire about its diameter and tangent at its extremity.

Q.3 Prove that the momental ellipsoid of a point on the rim of a hemisphere is

$2x^2 + 7(y^2 + z^2) - \frac{15}{4}xz = \text{constant}.$

UNIT – II

Q.4 A solid homogeneous cone, of height h and vertical angle 2α, oscillates about a horizontal axis through its vertex. Show that the length of the simple equivalent pendulum is $\frac{h}{5}(4 + \tan^2\alpha)$.

Q.5 A rod, of length 2a revolves with uniform angular velocity to about a vertical axis through a smooth joint at one extremity of the rod so that it describes a cone of semi-vertical angle α, we have $\omega^2 = 3g/(4a \cos \alpha)$. Prove that direction of reaction at the hinge makes with the vertical an angle $\tan^{-1}\left[\left(\frac{3}{4}\right)\tan\alpha\right]$.

UNIT – III

Q.6 A uniform solid cylinder is placed with its axis horizontal on a plane, whose inclination to the horizon is α. Show that the least coefficient of the friction between it & (and) the plane, so that it may roll and not slide, is $\left(\frac{1}{3}\right)$ tan α.

Q.7 Two equal uniform rods, AB & AC are freely jointed at A, and are placed on a smooth table so as to be a right angles. The rod AC is struck by a blow at C in a direction perpendicular to itself, show that the resulting velocities of the middle points of AB and AC are in the ratio 2 : 7.

UNIT – IV

Q.8 A uniform rod, of length 2a, is placed with one end in contact with a smooth horizontal table and is then allowed to fall; if α be its initial inclination to the vertical, show that its angular velocity, when it is inclined at an angle θ, is $\left\{\frac{6g}{a} \cdot \frac{\cos\alpha - \cos\theta}{1 + 3\sin^2\theta}\right\}^{\frac{1}{2}}$

Q.9 A circular ring of mass M and radius a, lies on a smooth horizontal plane, and an insect of mass m, resting on it starts and walks round it with uniform velocity ν relative to the ring. Show that the centre of the ring describes a circle with angular velocity.

$\omega = \frac{\nu}{a} \cdot \frac{m}{M + 2m}$

UNIT –V

Q.10 When the lagrangian function has the form $L = \dot{q}_k q_k - \sqrt{1 - \dot{q}_k}$, show that the generalized acceleration is zero?

Q.11 A heavy uniform rod of mass m and length 2a rotating in a vertical plane falls and strikes a smooth inelastic horizontal plane. If u & ω be its linear and angular velocities and θ be the inclination of the rod to the vertical just before the impact, Prove that the impulse J is given by $(1 + 3\sin^2\theta)\ J = m\ (u + a\ \omega \sin\theta)$.

PART – C

Q.12 Find the moment of inertia of the area of lemniscate $r^2 = a^2 \cos 2\theta$.

(i) about its axis.

(ii) about a line through the origin & perpendicular to its plane.

Q.13 A rod, of length 2a is suspended by a string of length ℓ, attached to one end, if the string and rod revolve about the vertical with uniform angular velocity & their inclinations to the vertical be θ and ϕ respectively, show that-

$$3\ell(\tan\phi - \tan\theta)\sin\theta = (4\tan\theta - 3\tan\phi)\, a \sin\phi$$

Q.14 An imperfectly rough sphere moves from rest down a plane inclined at an angle α to the horizon; discuss the motion.

Q.15 An elliptic lamina is rotating about its centre on a smooth horizontal plane. If ω_1, ω_2, ω_3 be its angular velocities when the extremity of its major axis, its focus and the extremity of its minor axis respectively become fixed; prove that $\frac{7}{\omega_1} = \frac{6}{\omega_2} + \frac{5}{\omega_3}$.

Q.16 A perfectly rough sphere lying inside a hollow cylinder, which rests on a perfectly rough plane, is slightly displaced from its position of equilibrium. Show that the time of a small oscillation is $2\pi\sqrt{\left(\frac{a-b}{g} \cdot \frac{14\,M}{10\,M+7m}\right)}$

where a is the radius of the cylinder, b that of the sphere, and M, m are the masses of the cylinder and sphere respectively.
