Roll No. ..... Total Pages: 07

2702

B.C.A. SECOND YEAR EXAMINATION, 2019

DISCRETE MATHEMATICS

Time: Three Hours

Maximum Marks: 100

Answers of all the questions (short answer as well as descriptive) are to be given in the main answer-book only. Answers of short answer type questions must be given in sequential order. Similarly, all the parts of one question of descriptive part should be answered at one place in the answer-book. One complete question should not be answered at different places in the answer-book. Write your roll number on question paper before starting to write the answers of questions.

Question paper consists of three parts.

All THREE parts are compulsory.

PART – A *[Marks: 20]*

(Very Short Answer)

Consists 10 questions of ***two marks*** *each.*

Maximum limit for each question is upto ***40*** *words.*

PART – B *[Marks: 20]*

(Short Answer)

Consists 5 questions of ***four marks*** *each.*

Maximum limit for each question is upto ***80*** *words.*

PART – C *[Marks: 60]*

(Long Answer)

Consists 5 questions of ***twelve marks*** *each with internal choices.*

PART – A

Q.1 (i) Write Binomial theorem for positive integral indices.

(ii) Explain Recurrence relation with suitable example.

(iii) Define Power set with example.

(iv) Explain Cartesian product of two sets.

(v) Show that P V (~ P) is a tautology.

(vi) Define types of Propositions.

(vii) Define Directed and Undirected graphs.

(viii) Let V ≡ {a, b, c, d, e} and

E ≡{(a, b), (a, c), (a, d), (b, c), (b, e), (e, d), (c, e)}

Find the degree of each of the vertices & also find the number of edges.

(ix) Explain Rooted tree.

(x) Write the properties of Binary tree.

PART – B

Q.2 Prove that "$2^n > n^2$ if $n \geq 5$".

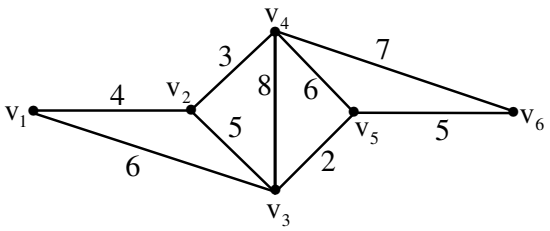
Q.3 Let the function $f : R \to R$ be defined by $f(x) = x^2 + 3$ and $g : R \to R$ be defined by $g(x) = \frac{x}{x+1}$.

Prove that $fog \neq gof$.

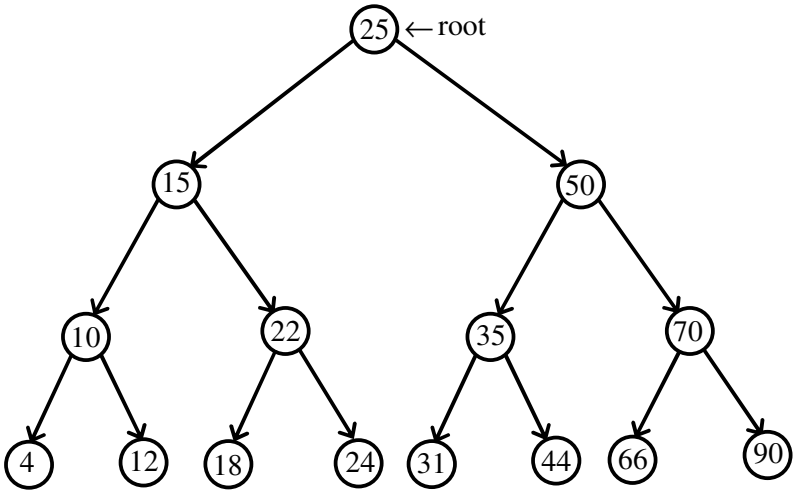
Q.4 Examine the validity of the following agreement:

'If it rains tomorrow then I shall bring my umbrella, provided it is repaired, it will rain tomorrow but my umbrella is not repaired, therefore I shall not bring my umbrella tomorrow".

Q.5 Find the shortest path from v_1 to v_6 in the following graph.



Q.6 Find the Pre-order, Inorder, Post-order traversal of Binary tree.



Also determine Height of tree and number of vertex at each level.

PART – C

Q.7 (a) Show that :

$P(n) = 1^2 + 2^2 + \ldots\ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}, n \geq 1$

by mathematical induction.

(b) Convert $(1101010.10101)_2$ to its decimal equivalent.

OR

Q.7 (a) Solve the recurrence relation

$a_r = 3a_{r-1} + 2, r \geq 1$ with

B.C. $a_0 = 1$

(b) Find the generating function for the Fibonacci sequence.

Q.8 If A & B are any two sets, then prove that-

(a) $A - B = A \Leftrightarrow A \cap B = \phi$

(b) $A - B = \phi \Leftrightarrow A \subseteq B$

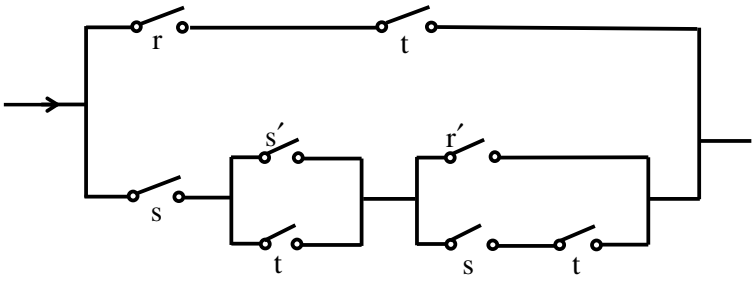
OR

Q.8 In the set of integers z = A, a relation R is defined by aRb as $a \equiv b \pmod 4$, show that-

(a) R is an equivalence relation

(b) Determine $\frac{A}{R}$.

Q.9 (a) Convert the following given circuit into a simplified circuit:



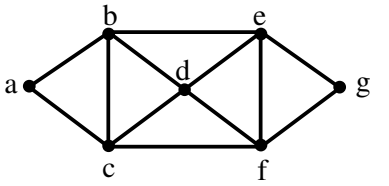
(b) Prove that $\sqrt{2}$ is an irrational number.

OR

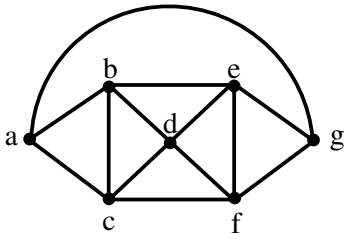
Q.9 (a) Prove that in a Boolean algebra B, for any two elements a, b ∈ B, a + b is lub (least upper bound), a.b is glb (greatest lower bound).

(b) Prove Demorgan's law of Propositions using truth table.

Q.10 (a) Explain colouring of graph and what are the chromatic numbers of the graph G & H as shown in the following figure-



G



H

(b) Prove that for any connected planar graph v – e + r = 2
where v, e and r are the number of vertices, edges and regions of the graph respectively.

OR

Q.10 (a) If a graph G = (V, E) is defined by V = $\{v_1, v_2, v_3, v_4, v_5\}$

E = $\{(v_1, v_2), (v_1, v_5), (v_2, v_3), (v_2, v_4), (v_3, v_4), (v_3, v_5), (v_4, v_5)\}$

$|V| = 5$, $|E| = 7$

Draw the graph and find the adjacency and incidence matrix of the graph G.

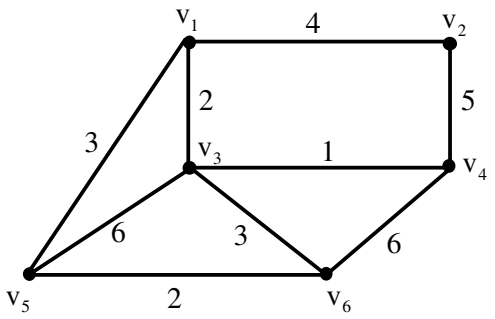
(b) Write short notes on:

(i) Planar graph

(ii) Isomorphic graph

(iii) Composition of graph

Q.11 (a) Find the minimal spanning tree for the following graph G using algorithms due to Kruskal and Prim.



(b) Show that a tree with n vertices, has precisely (n - 1) edges.

OR

Q.11 (a) If h is the height of a balanced complete binary tree on n vertices, then prove that-

$$h = \log_2\left(\frac{n+1}{2}\right)$$

(b) Define the following:

(i) Rooted tree

(ii) Tree of an algebraic expression

(iii) In order and Post order traversal
