Roll No. .... Total Pages: 04

6223

M.SC. MATHEMATICS IST SEMESTER EXAMINATION, 2019

Paper – III

Differential Equations and Calculus of Variations

Time: Three Hours

Maximum Marks: 80

PART – A (खण्ड – अ) *[Marks: 20]*

Answer all questions (50 words each).

All questions carry equal marks.

सभी प्रश्न अनिवार्य हैं। प्रत्येक प्रश्न का उत्तर 50 शब्दों से अधिक न हो।

सभी प्रश्नों के अंक समान हैं।

PART – B (खण्ड – ब) *[Marks: 40]*

Answer five questions (250 words each).

Selecting one from each unit. All questions carry equal marks.

*प्रत्येक इकाई से **एक–एक** प्रश्न चुनते हुए, कुल पाँच प्रश्न कीजिए।*

प्रत्येक प्रश्न का उत्तर 250 शब्दों से अधिक न हो।

सभी प्रश्नों के अंक समान हैं।

PART – C (खण्ड – स) *[Marks: 20]*

Answer any two questions (300 words each).

All questions carry equal marks.

*कोई **दो प्रश्न** कीजिए। प्रत्येक प्रश्न का उत्तर 300 शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – A

Q.1 (i) Write the Lipschitz condition.

(ii) Write general linear partial differential equation of second order.

(iii) Write the conditions for the equations $Rr + Ss + Tt + f(x, y, z, p, q) = 0$ for which it is of Hyperbolic, Parabolic and Elliptic type.

(iv) Classify the differential equation $y^2 \frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} + 4u = 0$.

(v) Define Orthogonal functions.

(vi) Define Rayleigh quotient.

(vii) Write three dimensional wave equation in Cartesian coordinates.

(viii) Define Dirac Delta function.

(ix) Define Functionals.

(x) Write Euler-Lagrange equation for calculus of variation.

PART – B

UNIT –I

Q.2 Test, whether the function defined below satisfies the Lipschitz condition or not $f(x, y) = x \sin y + y \cos x$ on the region $s: |x| \leq 1, |y| < \infty$. Also find Lipschitz constant.

Q.3 Test, whether the function defined below satisfies the Lipschitz condition or not $f(x, y) = y^2 + \cos x^2$ on the region $s: |x| \leq \frac{1}{2}; |y| \leq a$.

UNIT –II

Q.4 Solve $pt - qs = q^3$ by Monge's method.

Q.5 Reduce the equation $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$ to canonical form and hence solve it.

UNIT –III

Q.6 Show that eigen values of Sturm-Liouville system are real.

Q.7 Find the solution of Sturm-Liouville problem $y'' + \frac{1}{x}y' + \frac{\lambda}{x^2}y = 0,\ 1 \le x \le 2$ with boundary conditions $y(1) = 0,\ y(2) = 0$.

UNIT –IV

Q.8 By separation of variables, show that are dimensional wave equation $\frac{\partial^2 z}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 z}{\partial t^2}$ has the solution of the form A exp. $(\pm\, i\, nx\ \pm\ i\, nct)$, where A and n are constants.

Q.9 Show that the D'Alemberts' solution of the wave equation $\frac{\partial^2 \phi}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2}$ is $\phi = f(x + ct) + g(x - ct)$.

UNIT –V

Q.10 Find the path on which a particle, in the absence of friction, will slide from one fixed point to another point in the shortest time under the action of gravity.

Q.11 Find the curve joining the points (x_1, y_1) and (x_2, y_2) that yields a surface of revolution of minimum area when revolved about the axis. Also find the external of the functional.

PART – C

Q.12 Show that the differential equation $\frac{dy}{dx} = 1 + y + y^2 \cos x;\ y(0) = 0$ has a unique solution.

Q.13 Reduce the equation $\frac{\partial^2 z}{\partial x^2} + 4\frac{\partial^2 z}{\partial x\, \partial y} + 3\frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 9$ to canonical form and solve it.

Q.14 Obtain the formal expansion of the function f(x), where $f(x) = \pi x - x^2, 0 \leq x \leq \pi$ in the series of orthonormal characteristics function $\{\phi_n(x)\}$ of the Sturm-Liouville problem. $y'' + \lambda y = 0;\ y(0) = 0,\ y(\pi) = 0.$

Q.15 Find the Green's function for the equation $\frac{d^2y}{dx^2} - y = f(x), 0 \leq x \leq 1$ with the boundary conditions $y(0) = 0, y'(1) = 0.$

Q.16 Determine the external of the functional $I[y(x)] = \int_0^{\frac{\pi}{2}} (y''^2 - y^2 + x^2)\, dx$ that satisfies the conditions $y(0) = 1,\ y'(0) = 0, y\left(\frac{\pi}{2}\right) = 0, y'\left(\frac{\pi}{2}\right) = -1.$
