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M.Sc. MATHEMATICS IST SEMESTER EXAMINATION, 2019

Paper – IV

MECHANICS - I

Time: Three Hours

Maximum Marks: 80

PART – A (खण्ड – अ) *[Marks: 20]*

Answer all questions (50 words each).

All questions carry equal marks.

सभी प्रश्न अनिवार्य हैं। प्रत्येक प्रश्न का उत्तर 50 शब्दों से अधिक न हो।

सभी प्रश्नों के अंक समान हैं।

PART – B (खण्ड – ब) *[Marks: 40]*

Answer five questions (250 words each).

Selecting one from each unit. All questions carry equal marks.

*प्रत्येक इकाई से **एक–एक** प्रश्न चुनते हुए, कुल **पाँच** प्रश्न कीजिए।*

प्रत्येक प्रश्न का उत्तर 250 शब्दों से अधिक न हो।

सभी प्रश्नों के अंक समान हैं।

PART – C (खण्ड – स) *[Marks: 20]*

Answer any two questions (300 words each).

All questions carry equal marks.

*कोई **दो प्रश्न** कीजिए। प्रत्येक प्रश्न का उत्तर 300 शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – A

Q.1 (i) What is difference between Lagrange's and Euler's methods of motion?

(ii) Define boundary surface.

(iii) Write equation of motion of the fluid along r direction.

(iv) What is principle of permanence of irrotational motion?

(v) Define central orbit.

(vi) Write Kepler's third law of motion.

(vii) Why is a second plate P_2 mounted parallel to P_1 in Michelson and Morley experiment?

(viii) What is Lorentz- Fitzgerald contraction hypothesis?

(ix) Write formula between energy and momentum.

(x) What is difference between Newtonian and Hamiltonian formalisation.

PART – B

UNIT –I

Q.2 Obtain equation of continuity by Lagrange's approach.

Q.3 Show that $\frac{x^2}{a^2}e^t + \frac{y^2}{b^2}\cos t + \frac{z^2}{c^2}e^{-t}\sec t = 1$ is a possible form for the boundary surface of a liquid.

UNIT –II

Q.4 State and prove Bernoulli's theorem.

Q.5 Air obeying Boyle's law, is in motion in a uniform tube of small section. Prove that if ρ be the density and ν the velocity at a distance x from a fixed point at a time t, then-

$$\frac{\partial^2\rho}{\partial t^2} = \frac{\partial^2}{\partial x^2}\left[(\nu^2 + k)\rho\right].$$

UNIT –III

Q.6 A particle moves in an ellipse under a force which is always directed towards its focus, find the law of force and velocity at any point of its path.

Q.7 A particle describes an ellipse as a central orbit about the focus, prove that the velocity at the end of the minor axis is a geometric mean between the velocities at the end of any diameter.

UNIT –IV

Q.8 Explain emission theory.

Q.9 Obtain relativistic composition of parallel velocity under Lorentz transformations.

UNIT –V

Q.10 Obtain relativistic transformation formula for mass.

Q.11 Obtain relativistic Hamiltonian.

PART – C

Q.12 Show that if the velocity potential of an irrotational fluid motion is equal to

$A(x^2 + y^2 + z^2)^{-3/2} z \tan^{-1}\frac{y}{x}$,

the lines of flow will be on the series of surfaces-

$(x^2 + y^2 + z^2) = c^{2/3}(x^2 + y^2)^{2/3}$.

Q.13 An infinite mass of homogeneous incompressible fluid is at rest subject to a uniform pressure π and contains a spherical cavity of radius a, filled with gas at a pressure $m\pi$, prove that if the inertia of gas be neglected and Boyle's law be supposed to hold throughout the ensuing motion, the radius of the sphere will oscillate between the values a and na, where n is determined by the equation $1+3m \log n - n^3 = 0$.

Q.14 P particle under a central acceleration $\mu\mu^3(3 + 2a^2\mu^2)$, is projected from a distance a at an angle $tan^{-1}\frac{1}{2}$ with it with a velocity equal to that in a circle at the same distance; prove that the path is r = a tanθ.

Q.15 Obtain relativistic transformation formula for velocities using Lorentz transformation equations.

Q.16 An electron A moving with velocity μ_1 relative to an inertial frame in which another election B is at rest, strikes the election B. If after collision their directions of motion make angles θ and ϕ with the original direction of motion of A, Prove that-

$$\tan\theta \tan\phi = \frac{2}{r_1+1}$$

with $r_1^2\left(1 - \frac{\mu_1^2}{c^2}\right) = 1$
