Roll No. .... Total Pages: 04

7221

M.Sc. II[nd] SEMESTER EXAMINATION, 2019
MATHEMATICS
Paper – I
Algebra - II

Time: Three Hours

Maximum Marks: 80

PART – A (खण्ड – अ) *[Marks: 20]*

*Answer all questions (**50** words each).*

All questions carry equal marks.

*सभी प्रश्न अनिवार्य हैं। प्रत्येक प्रश्न का उत्तर **50** शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – B (खण्ड – ब) *[Marks: 40]*

*Answer **five** questions (**250** words each),*

selecting one from each unit. All questions carry equal marks.

*प्रत्येक इकाई से **एक–एक** प्रश्न चुनते हुए, कुल पाँच प्रश्न कीजिए।*

*प्रत्येक प्रश्न का उत्तर **250** शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – C (खण्ड – स) *[Marks: 20]*

*Answer any **two** questions (**300** words each).*

All questions carry equal marks.

*कोई **दो प्रश्न** कीजिए। प्रत्येक प्रश्न का उत्तर **300** शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – A

Q.1 (i) Define prime and irreducible elements.

(ii) Prove that the polynomial $x^2 + x + 1 \in z_2 (x)$ is irreducible.

(iii) Define Module.

(iv) If A and B are sub modules of an R - module M, then A + B is also module of M.

(v) Define Finitely Generated Modules.

(vi) Explain Noetherian Module.

(vii) Let F be a field with 5^{12} elements. Then find the total number of proper subfield of F.

(viii) Define normal field extension.

(ix) Explain automorphism of a field.

(x) Define solvability by radicals.

PART – B

UNIT – I

Q.2 Show that the ring Z of integers is a Euclidean ring.

Q.3 If a and b are any non-zero elements of a Euclidean ring R, then-

(i) b is a unit of R $\Rightarrow$ d (ab) = d (a)

(ii) b is not a unit of R $\Rightarrow$ d (ab) > d(a)

UNIT – II

Q.4 If M_1 and M_2 are two sub modules of an R – module M, then $M_1 \cap M_2$ is also a sub module of M.

Q.5 If $f : M \rightarrow M'$, then show that F is an Epimorphism if and only if $Im(f) = M'$.

UNIT – III

Q.6 If M is generated by $A = \{a_1, a_2, ..., a_n\}$ and A is L.I., then $M = A_1 \oplus A_2 \oplus .... \oplus An$, where A_i is a cyclic sub module generated by a_i.

Q.7 Show that any finite abelian group is the direct product of cyclic groups.

UNIT – IV

Q.8 Let K be a field extension of a field. Show that an element a is algebraic over F iff F(a) is a finite extension of F.

Q.9 Prove that decomposition fields are algebraic extension.

UNIT –V

Q.10 Let K be an extension of a field F, then the set G (K, F) of all automorphisms of K which leave every elements of F fixed is a subgroup of the group A(K) of all automorphisms of K.

Q.11 The multiplicative group of a Galois field or finite field is cyclic.

PART – C

Q.12 If a is non-zero non unit element of Euclidean ring R such that $a = p_1\ p_2\ .....\ p_m = q_1\ q_2\ ....\ q_n$ where each p_i and q_i is prime element of R. Then show that m = n and p_i is an associate of some q_i and each q_i is an associate of some p_i.

Q.13 Let M be an R – module and A be any sub-module of M. let $t : M \rightarrow M'$ be a linear transformation on R – module M to M' and $P : M \rightarrow \frac{M}{A}$ be a natural morphism. If the Kernel of t contain A, then ∃ a unique R – linear transformation $t' : \frac{M}{A} \rightarrow M'$ such that $t' \circ p \equiv t$.

Q.14 Let R be a Euclidean Ring, then any finitely generated R – module, M is the direct sum of a finite number of cyclic modules.

Q.15 A polynomial of degree n over a field can have at most n roots in any extension field.

Q.16 If K is normal extension of field of F of characteristic O. then ∃ a one – to – one correspondence between the set of subfield of K which contain F and the set of subgroup of G (K, F).
