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M. Sc. IInd SEMESTER EXAMINATION, 2019

MATHEMATICS

Paper – II

Complex Analysis

Time: Three Hours

Maximum Marks: 80

PART – A (खण्ड – अ) *[Marks: 20]*

*Answer all questions (**50** words each).*

All questions carry equal marks.

*सभी प्रश्न अनिवार्य हैं। प्रत्येक प्रश्न का उत्तर **50** शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – B (खण्ड – ब) *[Marks: 40]*

*Answer **five** questions (**250** words each),*

selecting one from each unit. All questions carry equal marks.

*प्रत्येक इकाई से **एक–एक** प्रश्न चुनते हुए, कुल पाँच प्रश्न कीजिए।*

*प्रत्येक प्रश्न का उत्तर **250** शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – C (खण्ड – स) *[Marks: 20]*

*Answer any **two** questions (**300** words each).*

All questions carry equal marks.

*कोई **दो प्रश्न** कीजिए। प्रत्येक प्रश्न का उत्तर **300** शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – A

Q.1 (i) Define analytic function.

(ii) Define circle of convergence of power series.

(iii) Define Fixed points.

(iv) Define Inverse points.

(v) Define Riemann definition of Integration.

(vi) Explain Simply – Connected Region.

(vii) State Morera's theorem.

(viii) Write Cauchy's Inequality.

(ix) Define singularities and pole of an Analytic function.

(x) Write Cauchy's theorem of residue.

PART – B

UNIT –I

Q.2 If f(z) is an analytic function prove that-

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|R\ f(z)|^2 = 2\,.\,|f'(z)|^2$$

Q.3 Find the region of Convergence of the power series.

$$\sum_{n=1}^{\infty}\frac{(z+2)^{n-1}}{(n+1)^3\cdot 4^n}$$

UNIT –II

Q.4 Discuss the bilinear transformation

$$\omega = \frac{az+b}{cz+d}$$

Where a, b, c, d are complex constants.

Q.5 If $\omega = 2z + z^2$, prove that circle $| z | = 1$ corresponds to a cardioid in the ω - plane.

UNIT –III

Q.6 Find the value of Integral-

$$\int_0^{1+i} (x - y + i\,x^2)\, dz$$

(a) Along the straight line from z = 0 to z = 1 + i

(b) Along the real axis from z = 0 to z = 1, and then along a line parallel to imaginary axis from z = 1 to z = 1 + i.

Q.7 State and prove Cauchy's Integral formula.

UNIT –IV

Q.8 Prove that If a function f(z) is analytic for all finite values of z, and is bounded, is a constant.

Q.9 Obtain expansions for $\frac{(z-2)(z+2)}{(z+1)(z+4)}$

(i) When $|z| < 1$

(ii) When $1 < |z| < 4$

(iii) When $|z| > 4$

UNIT –V

Q.10 Show that $\int_0^{2\pi} \frac{d\,\theta}{2+\cos\theta} = \frac{2\pi}{\sqrt{3}}$

Q.11 Evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta} d\theta$.

PART – C

Q.12 Prove that the sum of a power series in the complex variable z is analytic in the interior of its circle of convergence.

Q.13 Show that $\omega = \frac{5 - 4z}{4z - 2}$

transforms the circle of $|z| = 1$ into a circle of radius unity in the ω - plane and find the centre of this circle.

Q.14 If f(z) is analytic within a circle C, given by $|z - a| = R$. If $|f(z)| \leq M$ on C, then

$$|f^n(a)| \leq \frac{M \,.\, n!}{R^n}$$

Q.15 Drive Poisson's Integral formula.

Q.16 Find the residue of $\frac{z^3}{(z-1)^4(z-2)(z-4)}$ at z = 1.
