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7223

M. Sc. IInd SEMESTER EXAMINATION, 2019

MATHEMATICS

Paper – III

Special Functions

Time: Three Hours

Maximum Marks: 80

PART – A (खण्ड – अ) *[Marks: 20]*

*Answer all questions (**50** words each).*

All questions carry equal marks.

*सभी प्रश्न अनिवार्य हैं। प्रत्येक प्रश्न का उत्तर **50** शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – B (खण्ड – ब) *[Marks: 40]*

*Answer **five** questions (**250** words each),*

selecting one from each unit. All questions carry equal marks.

*प्रत्येक इकाई से **एक–एक** प्रश्न चुनते हुए, कुल पाँच प्रश्न कीजिए।*

*प्रत्येक प्रश्न का उत्तर **250** शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – C (खण्ड – स) *[Marks: 20]*

*Answer any **two** questions (**300** words each).*

All questions carry equal marks.

*कोई **दो प्रश्न** कीजिए। प्रत्येक प्रश्न का उत्तर **300** शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – A

Q.1 (a) Write the Legendre's Duplication Formula.

(b) Determine whether x = 0 is an ordinary point or a regular singular point of the differential equation

$$2x^2\frac{d^2y}{dx^2} + 7x(x+1)\frac{dy}{dx} - 3y = 0$$

(c) Write the orthogonal property for Legendre function.

(d) Calculate the value of $P_2(x)$ from Rodrigues formula.

(e) Write the associated Legendre Equation.

(f) Write the series expansion of Legendre Polynomial.

(g) Show that $\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x), n \geq 0.$

(h) Write the integral representation of Bessel function.

(i) Write the Rodrigues formula for Hn(x).

(j) Write the series form for Laguerre polynomials.

PART – B

UNIT –I

Q.2 Solve the Bessel equation $x^2y'' + xy' + (x^2 - n^2)y = 0$ in series, taking 2n as non-integral.

Q.3 Prove that –

$${}_2F_1(-n, 1-\beta-n; \alpha; 1) = \frac{(\alpha+\beta-1)_{2n}}{(\alpha)_n(\alpha+\beta-1)_n}$$

UNIT –II

Q.4 Show that –

$$(1 - 2xt + t^2)^{\frac{-1}{2}} = \sum_{n=0}^{\infty} t^n Pn(x)$$

Q.5 Show that –

$$\int_{-1}^{1} x\,.\,Pn(x)Pn - 1(x)\,dx = \frac{2n}{(2n-1)(2n+1)}$$

UNIT –III

Q.6 Show that all the roots of Pn(x) = 0 are different.

Q.7 Show that –

$$(2m + 1)x\,P_m^k(x) = (m + 1)P_{m+1}^k(x) + m\,P_{m-1}^k(x) - k(2m + 1)\sqrt{(1 - x^2)}\,P_m^{k-1}(x).$$

UNIT –IV

Q.8 Show that –

$$\int_0^1 x\,J_n(\alpha x)\,J_n(\beta x)\,dx = 0, \text{ if } \alpha \neq \beta$$

$$= \frac{1}{2} J_{n+1}^2(\alpha), \text{ if } \alpha = \beta.$$

Where α and β are the roots of Jn(x).

Q.9 Show that

(i) $\cos x = J_0 - 2J_2 + 2J_4$............

(ii) $\sin x = 2J_1 - 2J_3 + 2J_5$...........

UNIT –V

Q.10 State and prove that Rodrigues Formula for Hn(x).

Q.11 Show that –

$$\int_0^{\infty} e^{-x} L_n(x)\,L_m(x)dx = \delta nm$$

Where δnm is the Kronecker delta.

PART – C

Q.12 If m is a positive integer, and | x | > 1, show that

$$2F_1\left(\frac{m+1}{2}, \frac{m+2}{2}; 1; -\frac{1}{x^2}\right) = \frac{(-1)^m x^{m+1}}{m!} \frac{d^m}{dx^m}\left\{\frac{1}{\sqrt{(1+x^2)}}\right\}$$

Q.13 Find solution near x = 0 of

$$x^2 \frac{d^2y}{dx^2} + (x + x^2)\frac{dy}{dx} + (x - 9)y = 0$$

Q.14 Prove that -

$$J_n J'_{-n} - J'_n \ J_{-n} = \frac{-2\delta n \, n\pi}{\pi}$$

Q.15 Show that -

$$Pn(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2 - 1)^n$$

Q.16 Show that -

(i) $H'n(x) = 2n \, Hn - 1(x)$

(ii) $2x \, Hn(x) = 2n \, Hn - 1(x) + Hn + 1(x)$

(iii) $H'n(x) = 2x \, Hn(x) - Hn + 1(x)$
