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M.Sc. IInd Semester EXAMINATION, 2018

MATHEMATICS

Paper – I

(Algebra-II)

Time: Three Hours
Maximum Marks: 80

PART – A (खण्ड – अ) *[Marks: 20]*

*Answer all questions (**50** words each).*

All questions carry equal marks.

*सभी प्रश्न अनिवार्य हैं। प्रत्येक प्रश्न का उत्तर **50** शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – B (खण्ड – ब) *[Marks: 40]*

*Answer five questions (**250** words each).*

*Selecting **one** from each unit. All questions carry equal marks.*

*प्रत्येक इकाई से **एक–एक** प्रश्न चुनते हुए, कुल **पाँच** प्रश्न कीजिए।*

*प्रत्येक प्रश्न का उत्तर **250** शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – C (खण्ड – स) *[Marks: 20]*

*Answer any **two** questions (**300** words each).*

All questions carry equal marks.

*कोई **दो** प्रश्न कीजिए। प्रत्येक प्रश्न का उत्तर **300** शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – A

Q 1. (i) Prove that the polynomial $(x^2 + 3)$ over the field R of real number is irreducible but it is reducible over the field c of complex number.

(ii) What do you mean by unique factorisation domain?

(iii) Define a sub module generated by a set.

(iv) Define a quotient module.

(v) Define a cyclic module.

(vi) Define Neotherion module.

(vii) Explain the field extension.

(viii) Define splitting field with suitable example.

(ix) What do you understand by Automorphism.

(x) State fundamental theorem of Galois Theory.

PART – B

UNIT –I

Q 2. In a Principal Ideal domain each pair of elements surely has a G. C. D.

Q 3. Prove that every Euclidean ring possesses unity element.

UNIT –II

Q 4. Show that the linear sum of two sub – modules of an R – module is also a sub – module of the same.

Q 5. If f be a homomorphism of an R – module M onto an R – module N with Ker f = w then prove $N \cong \frac{H}{W}$.

UNIT –III

Q 6. Prove An R – module M is neotherion iff every sub module of M is finitely generated.

Q 7. Prove that any unital irreducible R – module is cyclic.

UNIT –IV

Q 8. Show that every finite extension of a Perfect field is perfect.

Q 9. Find the degree of splitting field of $x^4 - 5x^2 + 6$ over Q rational numbers.

UNIT –V

Q 10. Let F be any subfield of K and A be any group of automorphism of K then fixed field of A is a subfield of K.

Q11. Let E/F be a finite extension such that E/F is both normal and separable then show that $\frac{E}{F}$ is a Galois extension.

PART – C

Q12. Show that every Euclidean ring is a Principal ideal domain.

Q13. If M be an R – module and W be a sub module of an R – module then prove that the set $\frac{M}{W}$ of all sets of W in M will be a R – module.

Q14. Show that for an R – module M following statement are equivalent.

(i) M is neotherion

(ii) Every sub module of M is finitely generated.

(iii) Every non empty set s of sub modules of M has a maximal element.

Q15. If K be an extension field over F and $\propto \in$ K be a algebraic element of degree n then $[F(\propto): F] = n$.

Q16. State and prove Abel's theorem.
