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M.Sc. MATHEMATICS IIIrd SEMESTER EXAMINATION, 2019

Paper - I

Numerical Analysis-I

Time: Three Hours

Maximum Marks: 80

PART – A (खण्ड – अ)   [Marks: 20]

Answer all questions (50 words each).

All questions carry equal marks.

सभी प्रश्न अनिवार्य हैं। प्रत्येक प्रश्न का उत्तर 50 शब्दों से अधिक न हो।

सभी प्रश्नों के अंक समान हैं।

PART – B (खण्ड – ब)   [Marks: 40]

Answer five questions (250 words each).

Selecting one from each unit. All questions carry equal marks.

प्रत्येक इकाई से **एक–एक** प्रश्न चुनते हुए, कुल पाँच प्रश्न कीजिए।

प्रत्येक प्रश्न का उत्तर 250 शब्दों से अधिक न हो।

सभी प्रश्नों के अंक समान हैं।

PART – C (खण्ड – स)   [Marks: 20]

Answer any two questions (300 words each).

All questions carry equal marks.

कोई **दो प्रश्न** कीजिए। प्रत्येक प्रश्न का उत्तर 300 शब्दों से अधिक न हो।

सभी प्रश्नों के अंक समान हैं।

PART – A

Q.1 (i) Define rate of convergence of an iterative method.

(ii) Define simple root of $f(x) = 0$ with an example.

(iii) Determine an interval of length one unit in which the negative real root, which is smallest in magnitude lies for the equation $9x^3 + 18x^2 - 37x - 70 = 0$.

(iv) Which iterative method has quadratic rate of convergence?

(v) Write iterative formula of Birge-Vieta method for the polynomial equation $P_n(x) = 0$

(vi) Define system of simultaneous linear equations.

(vii) Which method is better in Jacobi and Gauss-Seidal iterative methods for solving simultaneous linear equations? Give justification.

(viii) Can we use partial pivoting in Gauss-Jordan method?

(ix) When do we use the power method?

(x) Define eigen value and eigen vector of a matrix A.

PART – B

UNIT –I

Q.2 The multiple root ξ of multiplicity two of a equation $f(x) = 0$ is to be determined. We consider the multipoint method-

$$x_{k+1} = x_k - \frac{f\left[x_k + \frac{2f(x_k)}{f'(x_k)}\right]}{2f'(x_k)}$$

Show that the iteration method has third order rate of convergence.

Q.3 Write the short note on complex roots.

UNIT –II

Q.4 Find the interval in which the smallest positive root of the following equation lies-

$$x^3 - x - 4 = 0$$

Determine the root correct to two decimal places using the bisection method.

Q.5 Determine the initial approximation of equation

$$x^4 - x - 10 = 0$$

for finding the smallest positive root. Use this to find the root correct to three decimal

places with the Newton-Raphson method.

UNIT –III

Q.6 Using Bairstow's method obtain the quadratic factor of the following equation (perform

two iterations)

$$x^4 - 3x^3 + 20x^2 + 44x + 54 = 0 \qquad \text{with } (p, q) = (2, 2).$$

Q.7 Use the Birge-Vieta method to find a real root correct to three decimals of the following

equation:

$$x^3 - 11x^2 + 32x - 22 = 0, \qquad p = 0.5$$

UNIT –IV

Q.8 Find the inverse of the matrix-

$$\begin{bmatrix} 2 & -1 & 2 \\ -1 & 1 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

by the Cholesky method.

Q.9 Determine the inverse of the matrix-

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$$

Using the partition method and hence find the solution of the system of equations-

$$x_1 + x_2 + x_3 = 1$$

$$4x_1 + 3x_2 - x_3 = 6$$

$$3x_1 + 5x_2 + 3x_3 = 4$$

UNIT –V

Q.10 List out the basic properties of eigen values and eigen vectors of a matrix.

Q.11 Find the largest eigen value of a matrix-

$$A = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \end{bmatrix}$$

Using power method.

PART – C

Q.12 Find the rate of convergence of Secant method.

Q.13 Solve the following system of equations-

$$x_1 + x_2 - x_3 = 2$$

$$2x_1 + 3x_2 + 5x_3 = -3$$

$$3x_1 + 2x_2 - 3x_3 = 6$$

Using by Gauss-elimination method with partial pivoting.

Q.14 Solve the system of equations-

$$4x_1 + 2x_2 + x_3 = 4$$

$$x_1 + 3x_2 + x_3 = 4$$

$$3x_1 + 2x_2 + 6x_3 = 7$$

Using the Gauss-Seidal method upto three iterations using the initial approximation,

$x^{(0)} = [0.1, 0.8, 0.5]^T$.

Q.15 Determine the largest eigen value and the corresponding eigen vector of the matrix-

$$A = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 1 & 4 \end{bmatrix}$$

to three correct decimal places using the power method.

Q.16 What is the disadvantage of the Regula-Falsi method? Also, find the smallest positive

root of $x^4 - x - 10 = 0$, correct to three decimal places.