Roll No. .... Total Pages: 07

8224

M.Sc. MATHEMATICS IIIrd SEMESTER
EXAMINATION, 2019
Paper - IV
Optimization Techniques-I

Time: Three Hours

Maximum Marks: 80

PART – A (खण्ड – अ) *[Marks: 20]*

Answer all questions (50 words each).

All questions carry equal marks.

सभी प्रश्न अनिवार्य हैं। प्रत्येक प्रश्न का उत्तर 50 शब्दों से अधिक न हो।

सभी प्रश्नों के अंक समान हैं।

PART – B (खण्ड – ब) *[Marks: 40]*

Answer five questions (250 words each).

Selecting one from each unit. All questions carry equal marks.

*प्रत्येक इकाई से **एक–एक** प्रश्न चुनते हुए, कुल पाँच प्रश्न कीजिए।*

प्रत्येक प्रश्न का उत्तर 250 शब्दों से अधिक न हो।

सभी प्रश्नों के अंक समान हैं।

PART – C (खण्ड – स) *[Marks: 20]*

Answer any two questions (300 words each).

All questions carry equal marks.

*कोई **दो प्रश्न** कीजिए। प्रत्येक प्रश्न का उत्तर 300 शब्दों से अधिक न हो।*

सभी प्रश्नों के अंक समान हैं।

PART – A

Q.1 (i) Write advantage of Dual Simplex Method over simplex method.

(ii) What do you mean by bounded value LPP?

(iii) Discuss sensitivity analysis with respect to changes in the coefficients $a_{ij} \notin B$, where a_{ij} is the coefficients of non-basic variables.

(iv) Define Post-optimality analysis.

(v) Explain addition of the new variable to a given L.P.P.

(vi) Explain Effect of deletion of a constraint from a given L.P.P.

(vii) Define integer programming problems.

(viii) Explain Fractional cut and λ-cut.

(ix) Write applications of PERT/CPM techniques.

(x) Explain Total float.

PART – B

UNIT –I

Q.2 Solve the following problem by dual simplex method:

Min z = $2x_1 + x_2$,

subject to $3x_1 + x_2 \geq 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \geq 3$

and $x_1, x_2 \geq 0$

Q.3 Explain Bounded Value Algorithm.

UNIT –II

Q.4 Given the following linear programming problem:

$$\text{Max } z = 3x_1 + 5x_2 + 4x_3 ,$$
$$\text{subject to } \quad 2x_1 + 3x_2 \le 8$$
$$2x_2 + 5x_3 \le 10$$
$$3x_1 + 2x_2 + 4x_3 \le 15$$
$$\text{and } \quad x_1, x_2, x_3 \ge 0$$

Find the range over which b_2 can be changed maintaining the feasibility of the solution.

Q.5 Given the L.P.P.-

$$\text{Max } z = 3x_1 + 5x_2$$
$$\text{subject to } \quad 3x_1 + 2x_2 \le 18$$
$$x_1 \le 4$$
$$x_2 \le 6$$
$$\text{and } \quad x_1, x_2 \ge 0$$

Determine optimum solution to the L.P.P and discuss the Effect on the optimality of the solution when the objective function is change to $z = 3x_1 + x_2$.

UNIT –III

Q.6 Discuss sensitivity analysis with respect to addition of new constraints.

Q.7 Let the optimum simplex table for a maximization problem (with all constraints of '≤' type) be-

		C_j	5	12	4	0	-M
Basic variable	C_B	X_B	x_1	x_2	x_3	x_4	A_1
x_2	12	$\frac{8}{5}$	0	1	$\frac{-1}{5}$	$\frac{2}{5}$	$\frac{-1}{5}$
x_1	5	$\frac{9}{5}$	1	0	$\frac{7}{5}$	$\frac{1}{5}$	$\frac{2}{5}$
$z = 14\frac{1}{5}$			0	0	$\frac{3}{5}$	$\frac{29}{5}$	$M\frac{-2}{5}$

where x_4 is slack and a_1 an artificial variable. Let a new variable $x_5 \ge 0$ be introduced in the problem with a cost 30 assigned to it in the objective function. Also given that the coefficients of x_5 in the two constraints are 5 and 7 respectively.

Discuss the Effect of this addition of a variable on the optimality of the optimum solution to the given problem.

UNIT –IV

Q.8 Explain and write the steps of Branch and Bound algorithm for integer programming problem.

Q.9 Solve the following I.P.P. by Gomory's Method-

$$\text{Max } z = 2x_1 + 20x_2 - 10x_3$$

$$\text{subject to } 2x_1 + 20x_2 + 4x_3 \leq 15$$

$$6x_1 + 20x_2 + 4x_3 = 20$$

and $x_1, x_2, x_3 \geq 0$ and are all integers.

UNIT –V

Q.10 A project consist of a series or tasks labelled A, B….. H, I with the following relationships (W < X, Y means X & Y cannot start until W is completed; X, Y < W means W cannot start until both X & Y are completed). With this notation, construct the network diagram having the following constraints:

A < D, E ; B, D < F ; C < G ; G < H ; F, G < I

Find also the optimum time of completion of the project, when the time (in days) of completion of each task is as follows:

Task :	A	B	C	D	E	F	G	H	I
Time :	23	8	20	16	24	18	19	4	10

Q.11 Explain the following terms in project evaluation and review technique:

(a) Pessimistic time

(b) Optimistic time

(c) Most likely time

(d) Expected time

(e) Variance

PART – C

Q.12 Considered the parametric LPP-

Max z = $(3-6\lambda)x_1 + (2-2\lambda)x_2 + (5+5\lambda)x_3$

subject to $x_1 + 2x_2 + x_3 \le 430$

$3x_1 + 2x_3 \le 460$

$x_1 + 4x_2 \le 420$

and $x_1, x_2, x_3 \ge 0$

Perform the parametric analysis and identify all the critical values of the parameter λ.

Q.13 Given the L.P.P.-

Max z = $3x_1 + 4x_2 + x_3 + 7x_4$

subject to $8x_1 + 3x_2 + 4x_3 + x_4 \le 7$

$2x_1 + 6x_2 + x_3 + 5x_4 \le 3$

$x_1 + 4x_2 + 5x_3 + 2x_4 \le 8$

and $x_1, x_2, x_3, x_4 \ge 0$

Find the optimal solution of the L.P.P and compute the limit for a_{24} so that the new solution remains optimal feasible solution.

Q.14 Consider the L.P.P.-

$$\text{Max } z = x_1 + 2x_2$$

$$\text{subject to } -x_1 + x_2 \leq 1 \text{ ------------ } (1)$$

$$x_1 + x_2 \leq 2 \text{ ------------- } (2)$$

$$\text{and } x_1, x_2 \geq 0$$

(a) Find the optimal solution.

(b) Discuss the effect of deletion of constraint (1) on the optimality of solution.

Q.15 Use Branch and Bound technique to solve the following problem-

$$\text{Max } z = 3x_1 + 3x_2 + 13x_3$$

$$\text{subject to } -3x_1 + 6x_2 + 7x_3 \leq 8$$

$$6x_1 + (-3x_2) + 7x_3 \leq 8$$

$$0 \leq x_j \leq 5$$

and x_j are integers for j = 1, 2, 3.

Q.16 The following table shows their normal time and cost, crash time and cost for a project-

Job	Normal time (in days)	Cost (₹)	Crash time (in days)	Crash cost (₹)
(1-2)	6	1400	4	1900
(1-3)	8	2000	5	2800
(2-3)	4	1100	2	1500
(2-4)	3	800	2	1400
(3-4)	Dummy	-	-	-
(3-5)	6	900	3	1600
(4-6)	10	2,500	6	3500
(5-6)	3	500	2	800

Indirect cost for the project is ₹ 300 per day.

(i) Draw the network of the project.

(ii) What is normal duration cost of the project?

(iii) If all activities are crashed, what will be the project duration and corresponding cost?

(iv) Find the optimum duration and minimum project cost.
